Math Majors of America Tournament for High Schools

2018 Tiebreaker Test

Name: __________

ID: __________

Team: __________

– 75 minutes
– no calculators
– show work for credit

1. Daniel has an unlimited supply of tiles labeled "2" and "n" where n is an integer. Find (with proof) all the values of n that allow Daniel to fill an 8×10 grid with these tiles such that the sum of the values of the tiles in each row or column is divisible by 11.

2. Prove that if a triangle has integer side lengths and the area (in square units) equals the perimeter (in units), then the perimeter is not a prime number.

3. Suppose n points are uniformly chosen at random on the circumference of the unit circle and that they are then connected with line segments to form an $n-$gon. What is the probability that the origin is contained in the interior of this $n-$gon? Give your answer in terms of n, and consider only $n \geq 3$.

4. A sequence of integers $\{s_n\}$ is defined as follows: fix integers a, b, c, and d, then set $s_1 = a$, $s_2 = b$, and $s_n = cs_{n-1} + ds_{n-2}$ for all $n \geq 3$. Create a second sequence $\{t_n\}$ by defining each t_n to be the remainder when s_n is divided by 2018 (so we always have $0 \leq t_n \leq 2017$). Let $N = (2018^2)!$. Prove that $t_N = t_{2N}$ regardless of the choices of a, b, c, and d.