MMATHS **Name:** ____________ **Team:** ____________

2024 Individual Round

ID: ____________

1. ______
2. ______
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10. ______
11. ______
12. ______

– 75 minutes
– no calculators
– positive integer answers

1. Let $ab^2 = 126, bc^2 = 14, cd^2 = 128, da^2 = 12$. Find $\frac{bd}{ac}$.

2. Grant has a box with 6 red balls, 5 blue balls, 4 green balls, 3 yellow balls, 2 orange balls, and 1 purple ball. Grant selects 6 balls at random, without replacement. Let P be the probability that Grant selects six balls of different colors, and let Q be the probability that Grant selects six balls of the same color. What is $\frac{P}{Q}$?

3. Let $f(x)$ be a function, where if q is an integer, than $f(\frac{1}{q}) = q$, and if m and n are real numbers, $f(m \cdot n) = f(m) \cdot f(n)$. If $f(\sqrt{2})$ can be written as $\frac{\sqrt{a}}{b}$ where a is not divisible by the square of any prime and b is a positive integer, then what is $a + b$?

4. Let ABC be an equilateral triangle with side length 1. Then, let M be the midpoint of $\overline{BC}$. The area of all points within ABC that are closer to M than either A, B, or C can be expressed as the fraction $\frac{\sqrt{a}}{b}$ where a is not divisible by the square of any prime and b is a positive integer. Find $a + b$.

5. Two subsets are called *disjoint* if they do not share any common elements. Compute the number of ordered tuples (A, B, C), where A, B, and C are subsets (not necessarily distinct or non-empty) of $\{1, 2, 3, 4, 5\}$ such that A and B are disjoint and B and C are disjoint.

6. How many 7 digit numbers are there that satisfy the following?

- All digits are distinct digits from 1 - 7.
- The first digit (from the left) is divisible by 1.
- The two-digit number formed by the first two digits is divisible by 2.
- The three-digit number formed by the first three digits is divisible by 3.
- The four-digit number formed by the first four digits is divisible by 4.
- The five-digit number formed by the first five digits is divisible by 5.
- The six-digit number formed by the first six digits is divisible by 6.

7. The sum $\sum_{x=-5}^{5} \sum_{y=-5}^{5} \frac{2^x 3^y}{(1+2^x)(1+3^y)}$ can be expressed as a fraction $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

8. Let circle A have radius 9, and let circle B have radius 5 and be internally tangent to circle A. The largest radius r such that there are two circles with radius r that lie inside circle A, are externally tangent to each other, and externally tangent with Circle B can be expressed as a fraction $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

9. 2048 frogs are sitting in a circle and each have a \$1 bill. After each minute, each frog will independently give away each of their \$1 bills to either the closest frog to their left or the closest frog to their right with equal probability. If a frog has \$0 at the end of any given minute, then they will not give any money but may receive money. The expected number of frogs to have at least \$1 after 3 minutes can be denoted as a common fraction in the form $\frac{a}{b}$. Find $a + b$.

10. Find the sum of all prime numbers p such that $\binom{20242024p}{p} \equiv 2024 \pmod{p}$.

11. Let n be the least possible value of

$$\sqrt{x^2 + y^2 - 2x + 6y + 19} + \sqrt{x^2 + y^2 + 8x - 4y + 21}$$

Find n^2.

12. Let ABC be a triangle with $\angle A = 60°$ and orthocenter H. Let B' be the reflection of B over AC, C' be the reflection of C over AB, and A' be the intersection of BC' and $B'C$. Let D be the intersection of $A'H$ and BC. If $BC = 5$ and $A'D = 4$, then the area of $\triangle ABC$ can be expressed as $a\sqrt{b} + \sqrt{c}$, where a, b, and c are positive integers, and b and c are not divisible by the square of any prime. Find $a + b + c$.