

MMATHS 2023 Lightning Finals

Yale Math Competitions

Sample Question

The area of an equilateral triangle with sides of length 12 is how many times the area of an equilateral triangle with sides of length 8?

Sample Question

The area of an equilateral triangle with sides of length 12 is how many times the area of an equilateral triangle with sides of length 8?

Answer: $\frac{9}{4}$ or 2.25

Note: Answers can be in any form, not just integers. However, a correct answer must be in simplest form in order to receive credit.

Question 1

Alice and Bob are competing in a game. A random integer between 1 and 2023, inclusive is hidden in a box. Both players guess an integer, and the player closest to the number in the box wins. Alice guesses 1000. If Bob chooses the optimal guess, what is the probability he wins?

Question 1

Alice and Bob are competing in a game. A random integer between 1 and 2023, inclusive is hidden in a box. Both players guess an integer, and the player closest to the number in the box wins. Alice guesses 1000. If Bob chooses the optimal guess, what is the probability he wins?

Answer: $\frac{1023}{2023}$

Question 2

Find the sum of the divisors of 2023.

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Find the sum of the divisors of 2023.

Answer: 2456

Question 3

A recipe calls for 6 eggs and 12 cups of flour, and makes 24 servings.
How many servings can be made with 18 eggs and 24 cups of flour?

Question 3

A recipe calls for 6 eggs and 12 cups of flour, and makes 24 servings.
How many servings can be made with 18 eggs and 24 cups of flour?

Answer: 48

Question 4

A certain charger can charge 3 phone batteries in an hour. A faster charger, which charges twice as fast, can charge 2 laptop batteries in 90 minutes. In percent, how much bigger is a laptop battery than a phone battery?

Question 4

A certain charger can charge 3 phone batteries in an hour. A faster charger, which charges twice as fast, can charge 2 laptop batteries in 90 minutes. In percent, how much bigger is a laptop battery than a phone battery?

Answer: 350%

Question 5

A coin of radius three is uniformly randomly dropped onto a hexagonal table of side length $4\sqrt{3}$ such that the center of the coin lies on the table. What is the probability it lays entirely on the table?

Question 5

A coin of radius three is uniformly randomly dropped onto a hexagonal table of side length $4\sqrt{3}$ such that the center of the coin lies on the table. What is the probability it lays entirely on the table?

Answer: $\frac{1}{4}$ or .25

Question 6

Sterling Library has 12 study rooms, each of which can accommodate either 6 or 8 people. The maximum number of people that could enter the library and fit in the rooms is 74. How many of the rooms fit 6 people?

Question 6

Sterling Library has 12 study rooms, each of which can accommodate either 6 or 8 people. The maximum number of people that could enter the library and fit in the rooms is 74. How many of the rooms fit 6 people?

Answer: 11

Question 7

Out of Yale's students, 49% study economics and 51% study computer science (some possibly study both or neither). What is the difference, in percent, between the maximum and minimum possible number of students studying neither?

Question 7

Out of Yale's students, 49% study economics and 51% study computer science (some possibly study both or neither). What is the difference, in percent, between the maximum and minimum possible number of students studying neither?

Answer: 49%

Question 8

What is the area of the largest quarter-circle that can fit inside a circle with radius 1?

Question 8

What is the area of the largest quarter-circle that can fit inside a circle with radius 1?

Answer: $\pi/2$

Question 9

A square, triangle, circle, and a line are drawn in a plane. What is the largest possible number of points of intersection?

Question 9

A square, triangle, circle, and a line are drawn in a plane. What is the largest possible number of points of intersection?

Answer: 26

Question 10

Anya repeatedly flips a coin. On heads, she wins a dollar, and on tails, she loses a dollar. What is the probability that she ends up with the same amount of money as she started with after flipping the coin 2023 times?

Question 10

Anya repeatedly flips a coin. On heads, she wins a dollar, and on tails, she loses a dollar. What is the probability that she ends up with the same amount of money as she started with after flipping the coin 2023 times?

Answer: 0%

Question 11

Nine distinct beads are lined up in a row. How many ways can three beads be chosen so that no pair of them are adjacent?

Question 11

Nine distinct beads are lined up in a row. How many ways can three beads be chosen so that no pair of them are adjacent?

Answer: 35

Question 12

A regular pentagon $ABCDE$ and a regular triangle XYZ share the same center O . At the same instant, both polygons start rotating counterclockwise about O such that each polygon rotates at a constant rate of 180° divided by the number of sides it has. After how many minutes will both polygons be back in their starting positions?

Question 12

A regular pentagon $ABCDE$ and a regular triangle XYZ share the same center O . At the same instant, both polygons start rotating counterclockwise about O such that each polygon rotates at a constant rate of 180° divided by the number of sides it has. After how many minutes will both polygons be back in their starting positions?

Answer: 30

Question 13

In how many ways can 36 dollars be paid using only 10-dollar bills, 5-dollar bills, and 1-dollar bills, not necessarily using all three?

Question 13

In how many ways can 36 dollars be paid using only 10-dollar bills, 5-dollar bills, and 1-dollar bills, not necessarily using all three?

Answer: 20

Question 14

If a and b are real numbers satisfying $a - b = 3$ and $ab = 9$, what is $a^3 - b^3$?

Question 14

If a and b are real numbers satisfying $a - b = 3$ and $ab = 9$, what is $a^3 - b^3$?

Answer: 108

Question 15

A large bag contains 20 red balls and 20 green balls. Every turn, a ball is drawn from the bag. If the ball is red, it is taken out of the bag, and a green ball is put into the bag. If the ball is green, it stays in the bag, and another green ball is added to the bag. On the 460th draw, the final red ball is taken out of the bag, and a green ball is put in. At this point, how many green balls are in the bag?

Question 15

A large bag contains 20 red balls and 20 green balls. Every turn, a ball is drawn from the bag. If the ball is red, it is taken out of the bag, and a green ball is put into the bag. If the ball is green, it stays in the bag, and another green ball is added to the bag. On the 460th draw, the final red ball is taken out of the bag, and a green ball is put in. At this point, how many green balls are in the bag?

Answer: 480

Question 16

The two-digit integer \underline{AB} is prime and also one more than a multiple of 9. What is the maximum possible value of $A + B$?

Question 16

The two-digit integer \underline{AB} is prime and also one more than a multiple of 9. What is the maximum possible value of $A + B$?

Answer: 10

Question 17

Positive real numbers r and s satisfy $r^2 + 2rs = 15s^2$. What is $\frac{r^2+s^2}{rs}$?

Question 17

Positive real numbers r and s satisfy $r^2 + 2rs = 15s^2$. What is $\frac{r^2+s^2}{rs}$?

Answer: $\frac{10}{3}$

Question 18

A positive integer less than or equal to 64 is chosen uniformly randomly. What is the probability that it is either one more than a multiple of 7 or one more than a multiple of 9, but not both?

Question 18

A positive integer less than or equal to 64 is chosen uniformly randomly. What is the probability that it is either one more than a multiple of 7 or one more than a multiple of 9, but not both?

Answer: $\frac{7}{32}$

Question 19

At a party, the guests share a large pie. The guests take turns getting their share of the pie in the following manner: The first guest takes half of the pie, and each guest after the first takes half of what is left. After all the guests take their share, less than 0.1% of the pie remains. What is the minimum possible number of guests at the party?

Question 19

At a party, the guests share a large pie. The guests take turns getting their share of the pie in the following manner: The first guest takes half of the pie, and each guest after the first takes half of what is left. After all the guests take their share, less than 0.1% of the pie remains. What is the minimum possible number of guests at the party?

Answer: 10

Question 20

An equilateral triangle and a square have the same perimeter. If the area of the equilateral triangle is $64\sqrt{3}$, what is the area of the square?

Question 20

An equilateral triangle and a square have the same perimeter. If the area of the equilateral triangle is $64\sqrt{3}$, what is the area of the square?

Answer: 144

Question 21

What is the distance between the points $(-4, -7, -10)$ and $(5, 5, 10)$?

Question 21

What is the distance between the points $(-4, -7, -10)$ and $(5, 5, 10)$?

Answer: 25

Question 22

Two points are uniformly randomly chose in the interior of a cube with side length 2. What is the probability that the distance between those two points is less than 4?

Question 22

Two points are uniformly randomly chose in the interior of a cube with side length 2. What is the probability that the distance between those two points is less than 4?

Answer: 1

Question 23

Two cubes are glued together at a face to form a rectangular prism with surface area 250. What is the volume of the rectangular prism?

Question 23

Two cubes are glued together at a face to form a rectangular prism with surface area 250. What is the volume of the rectangular prism?

Answer: 250

Question 24

In regular octagon $ABCDEFGH$, diagonals \overline{AE} and \overline{BF} intersect at O .
What is the degree measure of $\angle AOB$?

Question 24

In regular octagon $ABCDEFGH$, diagonals \overline{AE} and \overline{BF} intersect at O .
What is the degree measure of $\angle AOB$?

Answer: 45

Question 25

What is the positive difference between the volume of a cylinder of radius 5 and height 9 and the volume of a cone with the same radius and height, in terms of π ?

Question 25

What is the positive difference between the volume of a cylinder of radius 5 and height 9 and the volume of a cone with the same radius and height, in terms of π ?

Answer: 150π

Question 26

When turned on, a laptop consumes battery at a rate of 1 percent every 5 minutes. When the laptop is turned off and plugged in, its battery is replenished at a rate of 1 percent every 2 minutes. The battery is currently at 50 percent. If the laptop is both turned on and plugged in, how long will it take to fully charge the laptop?

Question 26

When turned on, a laptop consumes battery at a rate of 1 percent every 5 minutes. When the laptop is turned off and plugged in, its battery is replenished at a rate of 1 percent every 2 minutes. The battery is currently at 50 percent. If the laptop is both turned on and plugged in, how long will it take to fully charge the laptop?

Answer: 125 minutes

Question 27

In a survey of 180 high school students, students could report liking math, liking science, neither, or both. 110 said they liked math and 100 said they liked science. However, 50 said that they did not like either subject. How many students reported liking both subjects?

Question 27

In a survey of 180 high school students, students could report liking math, liking science, neither, or both. 110 said they liked math and 100 said they liked science. However, 50 said that they did not like either subject. How many students reported liking both subjects?

Answer: 80

Question 28

In a certain city, gasoline costs \$1.50 per gallon. A gasoline tank that holds 60 gallons at capacity is currently 20 percent full. How much, in dollars, does it cost to completely fill the tank?

Question 28

In a certain city, gasoline costs \$1.50 per gallon. A gasoline tank that holds 60 gallons at capacity is currently 20 percent full. How much, in dollars, does it cost to completely fill the tank?

Answer: 72

Question 29

Blice and Aob are playing a coin game with one stack of 4 coins and one stack of 9 coins. The player whose turn it is must take any positive number of coins from the pile of their choice. The player who takes the last coin wins. Blice and Aob alternate turns, with Blice going first. How many coins must Blice take from one of the piles on the first turn to guarantee she will win?

Question 29

Blice and Aob are playing a coin game with one stack of 4 coins and one stack of 9 coins. The player whose turn it is must take any positive number of coins from the pile of their choice. The player who takes the last coin wins. Blice and Aob alternate turns, with Blice going first. How many coins must Blice take from one of the piles on the first turn to guarantee she will win?

Answer: 5

Question 30

A three digit positive integer \underline{ABC} has the following properties: A is one greater than B , C is one greater than A , B is prime, A is even, and C is a perfect square. What is the integer \underline{ABC} ?

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A three digit positive integer \underline{ABC} has the following properties: A is one greater than B , C is one greater than A , B is prime, A is even, and C is a perfect square. What is the integer \underline{ABC} ?

Answer: 879

Question 31

Kite $ABCD$ has $AB = AD = 7$ and $BC = DC = 5\sqrt{2}$. The measure of $\angle ABC$ is 135° . What is the length of the kite's longer diagonal?

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Kite $ABCD$ has $AB = AD = 7$ and $BC = DC = 5\sqrt{2}$. The measure of $\angle ABC$ is 135° . What is the length of the kite's longer diagonal?

Answer: 13

Question 32

What is the remainder when $1^3 + 2^3 + 3^3 + \dots + 2023^3$ is divided by 6?

Question 32

What is the remainder when $1^3 + 2^3 + 3^3 + \dots + 2023^3$ is divided by 6?

Answer: 4

Question 33

A cylindrical cake with radius 5 and height 15 sits upright on a table with one of its circular bases touching the table. It is then sliced in half with a horizontal cut, producing two smaller cylindrical halves. By what percent is the combined surface area of these two halves greater than the surface area of the original cake?

Question 33

A cylindrical cake with radius 5 and height 15 sits upright on a table with one of its circular bases touching the table. It is then sliced in half with a horizontal cut, producing two smaller cylindrical halves. By what percent is the combined surface area of these two halves greater than the surface area of the original cake?

Answer: 25

Question 34

In a certain number base b , it is known that $18_b \times 40_b = 660_b$. What is b ?

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In a certain number base b , it is known that $18_b \times 40_b = 660_b$. What is b ?

Answer: 13

Question 35

A number of cantaloupes grow in a rectangular field measuring 14 meters by 20 meters. The number of cantaloupes per square meter is uniform throughout the field, except for a 4 meter by 4 meter square grid in its northeast corner, where there are 3 times as many cantaloupes per square meter as the rest of the field. If there are 1320 cantaloupes growing outside the square grid, how many cantaloupes are inside the square grid?

Question 35

A number of cantaloupes grow in a rectangular field measuring 14 meters by 20 meters. The number of cantaloupes per square meter is uniform throughout the field, except for a 4 meter by 4 meter square grid in its northeast corner, where there are 3 times as many cantaloupes per square meter as the rest of the field. If there are 1320 cantaloupes growing outside the square grid, how many cantaloupes are inside the square grid?

Answer: 240

Question 36

Right now, the time on an analog clock is exactly 9:00 AM. Starting our count from now, what will the time be when the minute hand passes the hour hand for the fourth time, to the nearest minute?

Question 36

Right now, the time on an analog clock is exactly 9:00 AM. Starting our count from now, what will the time be when the minute hand passes the hour hand for the fourth time, to the nearest minute?

Answer: 1:05pm or 13:05

Question 37

In a bag, there are nine sticks, each measuring a distinct length between 1 and 9 inches, inclusive. On a given turn, a stick is drawn from the bag and placed on the floor, with all the sticks in the bag having an equal chance of being drawn. What is the minimum number of turns needed to guarantee that some three sticks on the floor can form a non-degenerate triangle?

Question 37

In a bag, there are nine sticks, each measuring a distinct length between 1 and 9 inches, inclusive. On a given turn, a stick is drawn from the bag and placed on the floor, with all the sticks in the bag having an equal chance of being drawn. What is the minimum number of turns needed to guarantee that some three sticks on the floor can form a non-degenerate triangle?

Answer: 6

Question 38

The chance of rain tomorrow is 30 percent. The chance of rain the day after tomorrow is 40 percent if it rains tomorrow, and 20 percent if it does not rain tomorrow. What is the percent chance that it rains on exactly one of the next two days?

Question 38

The chance of rain tomorrow is 30 percent. The chance of rain the day after tomorrow is 40 percent if it rains tomorrow, and 20 percent if it does not rain tomorrow. What is the percent chance that it rains on exactly one of the next two days?

Answer: 32

Question 39

There are 12 musicians and 15 mathematicians at a party. Every musician shakes hands once with every other musician, and every mathematician shakes hands once with every other mathematician. Also, every musician-mathematician pair shakes hands twice. How many total handshakes occur at the party?

Question 39

There are 12 musicians and 15 mathematicians at a party. Every musician shakes hands once with every other musician, and every mathematician shakes hands once with every other mathematician. Also, every musician-mathematician pair shakes hands twice. How many total handshakes occur at the party?

Answer: 531

Question 40

What is the sum of the distinct positive prime divisors of 786?

Question 40

What is the sum of the distinct positive prime divisors of 786?

Answer: 136

Question 41

Starting at the same spot along a circular track that has a length of 360 meters, Wendy and Xavier run in opposite directions along the track, both at a rate of 6 meters per second. After they start, they give each other a high five every time they pass each other. How many times will they high five in the first 10 minutes, counting the one at the 10-minute mark?

Question 41

Starting at the same spot along a circular track that has a length of 360 meters, Wendy and Xavier run in opposite directions along the track, both at a rate of 6 meters per second. After they start, they give each other a high five every time they pass each other. How many times will they high five in the first 10 minutes, counting the one at the 10-minute mark?

Answer: 20

Question 42

For any positive integer n , define $f(n) = \sum_{k=1}^n k$ and $g(n) = \sum_{k=1}^n k^3$. What is the value of $\frac{g(15)}{f(15)}$?

Question 42

For any positive integer n , define $f(n) = \sum_{k=1}^n k$ and $g(n) = \sum_{k=1}^n k^3$. What is the value of $\frac{g(15)}{f(15)}$?

Answer: 120

Question 43

A drawer contains 5 colors of socks, with exactly 3 socks of each color. A matching pair of socks consists of two socks having the same color. Sam reaches into the drawer and pulls out socks randomly one at a time. What is the minimum number of socks Sam must pull out to guarantee she ends up with two (disjoint) matching pairs?

Question 43

A drawer contains 5 colors of socks, with exactly 3 socks of each color. A matching pair of socks consists of two socks having the same color. Sam reaches into the drawer and pulls out socks randomly one at a time. What is the minimum number of socks Sam must pull out to guarantee she ends up with two (disjoint) matching pairs?

Answer: 8

Question 44

A regular polygon with n sides has angles measuring 177° . A regular polygon with m sides has angles measuring 175° . What is $n - m$?

Question 44

A regular polygon with n sides has angles measuring 177° . A regular polygon with m sides has angles measuring 175° . What is $n - m$?

Answer: 48

Question 45

At a pizza party, there are a new generation of Teenage Mutant Ninja Turtles spread across 9th, 10th, 11th, and 12th grade. Every turtle in 9th and 10th grade consumes 2 slices of pizza, while every turtle in 11th and 12th grade consumes 3 slices of pizza. If there are 200 total turtles split evenly between the four grades, how many slices of pizza are consumed by all the Teenage Mutant Ninja Turtles?

Question 45

At a pizza party, there are a new generation of Teenage Mutant Ninja Turtles spread across 9th, 10th, 11th, and 12th grade. Every turtle in 9th and 10th grade consumes 2 slices of pizza, while every turtle in 11th and 12th grade consumes 3 slices of pizza. If there are 200 total turtles split evenly between the four grades, how many slices of pizza are consumed by all the Teenage Mutant Ninja Turtles?

Answer: 500

Question 46

A *size n Turtle Cube* is a cube-shaped puzzle, consisting of a n -by- n -by- n cube of sub-cubes. The time that it takes Tracy the turtle to solve a size n Turtle Cube is proportional to the number of sub-cubes appearing on its surface. If it takes Tracy 28 seconds to solve a size 4 Turtle Cube, how many seconds does it take Tracy to solve a size 5 Turtle Cube?

Question 46

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Answer: 49

Question 47

Timberland the turtle is going shopping, and notices that a leather jacket in a store is normally priced at 50 dollars, but is on clearance and being sold at a 20% discount. Timberland comes back the next week during a sale, and notices that another 20% discount is applied to the jacket's clearance price. A week later, however, Timberland noticed that the sale ended and the jacket's price is increased by 40 percent of the sale price. What is the positive difference, in dollars, between the jacket's final and original prices?

Question 47

Timberland the turtle is going shopping, and notices that a leather jacket in a store is normally priced at 50 dollars, but is on clearance and being sold at a 20% discount. Timberland comes back the next week during a sale, and notices that another 20% discount is applied to the jacket's clearance price. A week later, however, Timberland noticed that the sale ended and the jacket's price is increased by 40 percent of the sale price. What is the positive difference, in dollars, between the jacket's final and original prices?

Answer: \$5.20

Question 48

Triangle ABC has $\angle B = 23^\circ$ and $\angle C = 38^\circ$. Points D and E are on side \overline{BC} so that $\angle DAE = 2\angle BAD$ and $\angle EAC = 2\angle DAE$. What is $\angle EAC$ in degrees?

Question 48

Triangle ABC has $\angle B = 23^\circ$ and $\angle C = 38^\circ$. Points D and E are on side \overline{BC} so that $\angle DAE = 2\angle BAD$ and $\angle EAC = 2\angle DAE$. What is $\angle EAC$ in degrees?

Answer: 68

Question 49

Each corner of square $ABCD$ is to be labelled with either 1, 2, or 3. In how many ways can this be done if the labels of the two vertices on each side of the square differ by no more than 1?

Question 49

Each corner of square $ABCD$ is to be labelled with either 1, 2, or 3. In how many ways can this be done if the labels of the two vertices on each side of the square differ by no more than 1?

Answer: 35

Question 50

A rhombus has sides of length 20 and a diagonal of length 24. What is its area?

Question 50

A rhombus has sides of length 20 and a diagonal of length 24. What is its area?

Answer: 384

Question 51

Two dark chocolate turtles and five white chocolate turtles together contain 194 calories, while four dark chocolate turtles and one white chocolate turtle together contain 145 calories. How many calories are in one white chocolate turtle?

Question 51

Two dark chocolate turtles and five white chocolate turtles together contain 194 calories, while four dark chocolate turtles and one white chocolate turtle together contain 145 calories. How many calories are in one white chocolate turtle?

Answer: 27

Question 52

A cone and a cylinder sit on a table. The cone has 6 times the cylinder's radius but $\frac{1}{2}$ the cylinder's height. What is the ratio of the volume of the cone to the volume of the cylinder?

Question 52

A cone and a cylinder sit on a table. The cone has 6 times the cylinder's radius but $\frac{1}{2}$ the cylinder's height. What is the ratio of the volume of the cone to the volume of the cylinder?

Answer: 6

Question 53

A rectangular prism has edges of length 2, 4, and 6. What is the distance from the center of the prism to one of its corner vertices?

Question 53

A rectangular prism has edges of length 2, 4, and 6. What is the distance from the center of the prism to one of its corner vertices?

Answer: $\sqrt{14}$

Question 54

There are 32 markers to be put in pencil cases. Each pencil case that is used must be filled to capacity. Large pencil cases hold 7 markers each, medium pencil cases hold 5 markers each, and small pencil cases hold 3 markers each. What is the minimum number of pencil cases required to hold all the markers?

Question 54

There are 32 markers to be put in pencil cases. Each pencil case that is used must be filled to capacity. Large pencil cases hold 7 markers each, medium pencil cases hold 5 markers each, and small pencil cases hold 3 markers each. What is the minimum number of pencil cases required to hold all the markers?

Answer: 6

Question 55

A car and a train both start at the same location and head toward a goal post that is one mile away. The train travels at a constant speed of 40 miles per hour, and the car travels at a constant speed of 60 miles per hour. How many seconds after the car passes the goal post will the train do the same?

Question 55

A car and a train both start at the same location and head toward a goal post that is one mile away. The train travels at a constant speed of 40 miles per hour, and the car travels at a constant speed of 60 miles per hour. How many seconds after the car passes the goal post will the train do the same?

Answer: 30

Question 56

Define the operation \star according to $a \star b = 3a^2 + kb^2$ for some constant k . It is known that $-2 \star 5 = 262$. What is k ?

Question 56

Define the operation \star according to $a \star b = 3a^2 + kb^2$ for some constant k . It is known that $-2 \star 5 = 262$. What is k ?

Answer: 10

Question 57

Evaluate $\frac{1}{4 + \frac{1}{3 + \frac{1}{2 + \frac{1}{1}}}}$.

Question 57

Evaluate $\frac{1}{4 + \frac{1}{3 + \frac{1}{2 + \frac{1}{1}}}}$.

Answer: $\frac{10}{43}$

Question 58

A tired college student with an exam the next day sets four alarms that go off in the morning. Starting from 8 o'clock, the first alarm sounds every 2 minutes, the second alarm every 4 minutes, the third alarm every 6 minutes, and the fourth alarm every 8 minutes. If the alarms all sound at 8 o'clock, when is the next time they will all sound?

Question 58

A tired college student with an exam the next day sets four alarms that go off in the morning. Starting from 8 o'clock, the first alarm sounds every 2 minutes, the second alarm every 4 minutes, the third alarm every 6 minutes, and the fourth alarm every 8 minutes. If the alarms all sound at 8 o'clock, when is the next time they will all sound?

Answer: 8 : 24

Question 59

If $\left(x \cdot \frac{x^2(x^3)^4}{x^5}\right)^6 = x^m$, what is the value of m ?

Question 59

If $\left(x \cdot \frac{x^2(x^3)^4}{x^5}\right)^6 = x^m$, what is the value of m ?

Answer: 60

Question 60

A quadrilateral has a pair of opposite vertices located at $(0, 0)$ and $(9, 0)$. Its other pair of opposite vertices are located at $(3, a)$ and $(6, b)$. If its area is 72, what is $a - b$?

Question 60

A quadrilateral has a pair of opposite vertices located at $(0, 0)$ and $(9, 0)$. Its other pair of opposite vertices are located at $(3, a)$ and $(6, b)$. If its area is 72, what is $a - b$?

Answer: 16

Question 61

What is the sum of the integers between -50 and 53 , inclusive?

Question 61

What is the sum of the integers between -50 and 53 , inclusive?

Answer: 156

Question 62

There are 4 red pencils and 4 blue pencils in a basket. In how many ways can the pencils be drawn from the basket, one at a time until the basket is empty, if pencils of the same color are indistinguishable?

Question 62

There are 4 red pencils and 4 blue pencils in a basket. In how many ways can the pencils be drawn from the basket, one at a time until the basket is empty, if pencils of the same color are indistinguishable?

Answer: 70

Question 63

What is the largest positive integer with 6 positive divisors such that all of its prime divisors are less than or equal to 5?

Question 63

What is the largest positive integer with 6 positive divisors such that all of its prime divisors are less than or equal to 5?

Answer: 3125

Question 64

Two circles O_1 and O_2 have radii r_1 and r_2 respectively. If a 74° arc on O_1 has the same length as a 111° arc on O_2 , what is the ratio of the area of O_1 to the area of O_2 ?

Question 64

Two circles O_1 and O_2 have radii r_1 and r_2 respectively. If a 74° arc on O_1 has the same length as a 111° arc on O_2 , what is the ratio of the area of O_1 to the area of O_2 ?

Answer: $\frac{9}{4}$

Question 65

There are four prime numbers between 100 and 110, inclusive. Find their sum.

Question 65

There are four prime numbers between 100 and 110, inclusive. Find their sum.

Answer: 420

Question 66

We construct the sequence $\{p_n\}_{n=1}^{\infty}$ as follows. Let $p_1 = 3$. If n is odd, then p_{n+1} is the smallest prime greater than p_n such that the remainder of p_{n+1} when divided by 6 is equal to 5. If n is even, then p_{n+1} is the smallest prime greater than p_n such that p_{n+1} divided by 6 gives a remainder of 1. Find the sum of all possible values of $p_n + p_{n+1}$ where $p_n + p_{n+1}$ is a power of a prime.

Question 66

We construct the sequence $\{p_n\}_{n=1}^{\infty}$ as follows. Let $p_1 = 3$. If n is odd, then p_{n+1} is the smallest prime greater than p_n such that the remainder of p_{n+1} when divided by 6 is equal to 5. If n is even, then p_{n+1} is the smallest prime greater than p_n such that p_{n+1} divided by 6 gives a remainder of 1. Find the sum of all possible values of $p_n + p_{n+1}$ where $p_n + p_{n+1}$ is a power of a prime.

Answer: 8

Question 67

Bob starts with the number 1, and multiplies by either 2 or 3 at a given step until he reaches 216. What is the sum of all the possible distinct numbers he could have at any step?

Question 67

Bob starts with the number 1, and multiplies by either 2 or 3 at a given step until he reaches 216. What is the sum of all the possible distinct numbers he could have at any step?

Answer: 600

Question 68

Three distinct prime numbers p , q , and r sum to 48. If $p < q < r$, what is the smallest possible value of $r - q$?

Question 68

Three distinct prime numbers p , q , and r sum to 48. If $p < q < r$, what is the smallest possible value of $r - q$?

Answer: 12

Question 69

Four siblings share 128 pieces of candy. The two youngest siblings each get half as many pieces of candy as the second oldest child, who in turn gets half as many pieces of candy as the oldest child. How many pieces of candy does the oldest child get?

Question 69

Four siblings share 128 pieces of candy. The two youngest siblings each get half as many pieces of candy as the second oldest child, who in turn gets half as many pieces of candy as the oldest child. How many pieces of candy does the oldest child get?

Answer: 64

Question 70

A triangle ABC has vertices at $A(0, 0)$, $B(3, 7)$, and $C(8, 2)$. The midpoints of \overline{BC} , \overline{CA} , and \overline{AB} are points $D(d_1, d_2)$, $E(e_1, e_2)$, and $F(f_1, f_2)$, respectively. What is $d_1 + d_2 + e_1 + e_2 + f_1 + f_2$?

Question 70

A triangle ABC has vertices at $A(0, 0)$, $B(3, 7)$, and $C(8, 2)$. The midpoints of \overline{BC} , \overline{CA} , and \overline{AB} are points $D(d_1, d_2)$, $E(e_1, e_2)$, and $F(f_1, f_2)$, respectively. What is $d_1 + d_2 + e_1 + e_2 + f_1 + f_2$?

Answer: 20

Question 71

If $\frac{b^2 - a^2}{c^2 - a^2} = 2$ and $b = 7a$, what is $\left|\frac{c}{a}\right|$?

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If $\frac{b^2 - a^2}{c^2 - a^2} = 2$ and $b = 7a$, what is $\left|\frac{c}{a}\right|$?

Answer: 5

Question 72

Find the number of positive divisors that 12^3 and 18^3 have in common.

Question 72

Find the number of positive divisors that 12^3 and 18^3 have in common.

Answer: 16

Question 73

An ammonia solution consists of 5 milliliters of ammonia mixed with 95 milliliters of water. How many milliliters of water must be added to the solution to reduce its ammonia concentration to 2 percent?

Question 73

An ammonia solution consists of 5 milliliters of ammonia mixed with 95 milliliters of water. How many milliliters of water must be added to the solution to reduce its ammonia concentration to 2 percent?

Answer: 150

Question 74

There are 40 fish in a pond. A fisherman catches 4 of them at random, tags them, and returns them to the pond. Later, a second fisherman arrives at the pond and catches 2 fish. What is the probability that both fish are tagged?

Question 74

There are 40 fish in a pond. A fisherman catches 4 of them at random, tags them, and returns them to the pond. Later, a second fisherman arrives at the pond and catches 2 fish. What is the probability that both fish are tagged?

Answer: $\frac{1}{130}$

Question 75

A car that gets 35 miles per gallon has a gas tank that can hold 12 gallons at capacity. The gas tank is currently half full. How many miles can the car travel with the amount of gas it currently has?

Question 75

A car that gets 35 miles per gallon has a gas tank that can hold 12 gallons at capacity. The gas tank is currently half full. How many miles can the car travel with the amount of gas it currently has?

Answer: 210

Question 76

For how many integers k between 1 and 100 inclusive is $\frac{k}{6}$ a fraction in simplest form (improper fractions are okay)?

Question 76

For how many integers k between 1 and 100 inclusive is $\frac{k}{6}$ a fraction in simplest form (improper fractions are okay)?

Answer: 33

Question 77

What is the remainder when $14 \cdot 12!$ is divided by 13?

Question 77

What is the remainder when $14 \cdot 12!$ is divided by 13?

Answer: 12

Question 78

What is the largest integer value of x such that $\sqrt{x^6 + 2x^3 + 1} - x^3 > 1$?

Question 78

What is the largest integer value of x such that $\sqrt{x^6 + 2x^3 + 1} - x^3 > 1$?

Answer: -2

Question 79

Define a sequence $(a_i)_{i \geq 1}$ by $a_1 = 1$ and $a_i = \frac{1}{i+a_{i-1}}$ for all $i \geq 2$. Find $\lim_{n \rightarrow \infty} a_n$.

Question 79

Define a sequence $(a_i)_{i \geq 1}$ by $a_1 = 1$ and $a_i = \frac{1}{i+a_{i-1}}$ for all $i \geq 2$. Find $\lim_{n \rightarrow \infty} a_n$.

Answer: 0

Question 80

How many real roots does the polynomial $(x - i)^6 - \frac{64}{27}$ have?

Question 80

How many real roots does the polynomial $(x - i)^6 - \frac{64}{27}$ have?

Answer: 2

Question 81

Find the remainder when the sum of the digits of 2024^{2023} is divided by 9.

Question 81

Find the remainder when the sum of the digits of 2024^{2023} is divided by 9.

Answer: 8

Question 82

Let G be a graph, and let S be the set of shortest paths between any pair of vertices in G . The diameter of a graph is the length of the longest path in S . What is the smallest possible diameter of a connected graph with 20 vertices?

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Let G be a graph, and let S be the set of shortest paths between any pair of vertices in G . The diameter of a graph is the length of the longest path in S . What is the smallest possible diameter of a connected graph with 20 vertices?

Answer: 1