Name: __________

ID : __________

Team : __________

— 75 minutes
— no calculators
— all answers are positive integers

Math Majors of America Tournament for High Schools

2020 Tiebreaker Test

1. A positive integer n is called an *untouchable number* if there is no positive integer m for which the sum of the factors of m (including m itself) is $n+m$. Find the sum of all of the untouchable numbers between 1 and 10 (inclusive).

2. Suppose that points A and B lie on circle Ω, and suppose that points C and D are the trisection points of major arc $\widehat{AB}$, with C closer to B than A. Let E be the intersection of line AB with the line tangent to Ω at C. Suppose that $DC = 8$ and $DB = 11$. If $DE = a\sqrt{b}$ for integers a and b with b squarefree, find $a+b$.

3. Let a, b be two real numbers such that

$$\sqrt[3]{a} - \sqrt[3]{b} = 10, \qquad ab = \left(\frac{8-a-b}{6}\right)^3.$$

Find $a-b$.

4. Define the function $f(n)$ for positive integers n as follows: if n is prime, then $f(n) = 1$; and $f(ab) = a \cdot f(b) + f(a) \cdot b$ for all positive integers a and b. How many positive integers n less than 5^{50} have the property that $f(n) = n$?

5. Let x, y be positive reals such that $x \neq y$. Find the minimum possible value of $(x+y)^2 + \frac{54}{xy(x-y)^2}$.

6. Consider the function $f(n) = n^2 + n + 1$. For each n, let d_n be the smallest positive integer with $\gcd(n, d_n) = 1$ and $f(n) \mid f(d_n)$. Find $d_6 + d_7 + d_8 + d_9 + d_{10}$.