Math Majors of America Tournament for High Schools

2018 Mixer Test

Name: ________

Team : ________

1. ________
2. ________
3. ________
4. ________
5. ________
6. ________
7. ________
8. ________
9. ________
10. ________
11. ________
12. ________

– 75 minutes
– no calculators
– simplify answers

1. Suppose $\frac{x}{y} = 0.\overline{ab}$ where x and y are relatively prime positive integers and $ab + a + b + 1$ is a multiple of 12. Find the sum of all possible values of y.

2. Let A be the set of points $\{(0, 0), (2, 0), (0, 2), (2, 2), (3, 1), (1, 3)\}$. How many distinct circles pass through at least three points in A?

3. Jack and Jill need to bring pails of water home. The river is the x-axis, Jack is initially at the point $(-5, 3)$, Jill is initially at the point $(6, 1)$, and their home is at the point $(0, h)$ where $h > 0$. If they take the shortest paths home given that each of them must make a stop at the river, they walk exactly the same total distance. What is h?

4. What is the largest perfect square which is not a multiple of 10 and which remains a perfect square if the ones and tens digits are replaced with zeroes?

5. In convex polygon $\mathcal{P}$, each internal angle measure (in degrees) is a distinct integer. What is the maximum possible number of sides $\mathcal{P}$ could have?

6. How many polynomials $p(x)$ of degree exactly 3 with real coefficients satisfy

$$p(0), p(1), p(2), p(3) \in \{0, 1, 2\}?$$

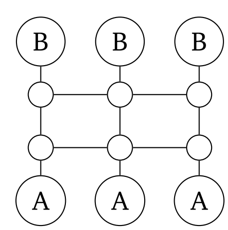
7. Six spheres, each with radius 4, are resting on the ground. Their centers form a regular hexagon, and adjacent spheres are tangent. A seventh sphere, with radius 13, rests on top of and is tangent to all six of these spheres. How high above the ground is the center of the seventh sphere?

8. You have a paper square. You may fold it along any line of symmetry. (That is, the layers of paper must line up perfectly.) You then repeat this process using the folded piece of paper. If the direction of the folds does not matter, how many ways can you make exactly eight folds while following these rules?

9. Quadrilateral $ABCD$ has $\overline{AB} = 40$, $\overline{CD} = 10$, $\overline{AD} = \overline{BC}$, $m\angle BAD = 20°$, and $m\angle ABC = 70°$. What is the area of quadrilateral $ABCD$?

10. We say that a permutation σ of the set $\{1, 2, \ldots n\}$ *preserves divisibilty* if $\sigma(a)$ divides $\sigma(b)$ whenever a divides b. How many permutations of $\{1, 2, ..., 40\}$ preserve divisibility? (A permutation of $\{1, 2, \ldots n\}$ is a function σ from $\{1, 2, \ldots n\}$ to itself such that for any $b \in \{1, 2, \ldots n\}$, there exists some $a \in \{1, 2, \ldots n\}$ satisfying $\sigma(a) = b$.)

11. In the diagram shown at right, how many ways are there to remove at least one edge so that some circle with an "A" and some circle with a "B" remain connected?



12. Let $\mathcal{S}$ be the set of the 125 points in three-dimension space of the form (x, y, z) where x, y, and z are integers between 1 and 5, inclusive. A family of snakes lives at the point $(1, 1, 1)$, and one day they decide to move to the point $(5, 5, 5)$. Snakes may slither only in increments of $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$. Given that at least one snake has slithered through each point of $\mathcal{S}$ by the time the entire family has reached $(5, 5, 5)$, what is the smallest number of snakes that could be in the family?