Name: __________

Team : __________

1. __________

2. __________

3. __________

4. __________

5. __________

6. __________

7. __________

8. __________

9. __________

10. __________

11. __________

12. __________

– 75 minutes
– no calculators
– simplify answers

Math Majors of America Tournament for High Schools

2018 Individual Test

1. Five friends arrive at a hotel which has three rooms. Rooms A and B hold two people each, and room C holds one person. How many different ways could the five friends lodge for the night?

2. The set of numbers $\{1, 3, 8, 12, x\}$ has the same average and median. What is the sum of all possible values of x? (Note that x is not necessarily greater than 12.)

3. How many four-digit numbers $\overline{ABCD}$ are there such that the three-digit number $\overline{BCD}$ satisfies $\overline{BCD} = \frac{1}{6}\overline{ABCD}$? (Note that A must be nonzero.)

4. Find the smallest positive integer n such that n leaves a remainder of 5 when divided by 14, n^2 leaves a remainder of 1 when divided by 12, and n^3 leaves a remainder of 7 when divided by 10.

5. In rectangle $ABCD$, let E lie on $\overline{CD}$, and let F be the intersection of $\overline{AC}$ and $\overline{BE}$. If the area of $\triangle ABF$ is 45 and the area of $\triangle CEF$ is 20, find the area of the quadrilateral $ADEF$.

6. If x and y are integers and $14x^2y^3 - 38x^2 + 21y^3 = 2018$, what is the value of x^2y?

7. A, B, C, D all lie on a circle with $\overline{AB} = \overline{BC} = \overline{CD}$. If the distance between any two of these points is a positive integer, what is the smallest possible perimeter of quadrilateral $ABCD$?

8. Compute

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{m\cos^2(n) + n\sin^2(m)}{3^{m+n}(m+n)}.$$

9. Diane has a collection of weighted coins with different probabilities of landing on heads, and she flips nine coins sequentially according to a particular set of rules. She uses a coin that always lands on heads for her first and second flips, and she uses a coin that always lands on tails for her third flip. For each subsequent flip, she chooses a coin to flip as follows: if she has so far flipped a heads out of b total flips, then she uses a coin with an $\frac{a}{b}$ probability of landing on heads. What is the probability that after all nine flips, she has gotten six heads and three tails?

10. For any prime number p, let S_p be the sum of all the positive divisors of $37^p p^{37}$ (including 1 and $37^p p^{37}$). Find the sum of all primes p such that S_p is divisible by p.

11. Six people are playing poker. At the beginning of the game, they have 1, 2, 3, 4, 5, and 6 dollars, respectively. At the end of the game, nobody has lost more than a dollar, and each player has a distinct nonnegative integer dollar amount. (The total amount of money in the game remains constant.) How many distinct finishing rankings (i.e. lists of first place through sixth place) are possible?

12. Let C_1 be a circle of radius 1, and let C_2 be a circle of radius $\frac{1}{2}$ internally tangent to C_1. Let $\{\omega_0, \omega_1, \ldots\}$ be an infinite sequence of circles, such that ω_0 has radius $\frac{1}{2}$ and each ω_k is internally tangent to C_1 and externally tangent to both C_2 and ω_{k+1}. (The ω_k's are mutually distinct.) What is the radius of ω_{100}?