Individual Round

Girls in Math at Yale

February 27, 2021

Problem 1 *Given that* $2x + 7y = 3$, *find* $2^{6x+21y-4}$.

Problem 2 *A box of strawberries, containing* 12 *strawberries total, costs* \$2. *A box of blueberries, containing* 48 *blueberries total, costs* \$3. *Suppose that for* \$12, *Sareen can either buy* m *strawberries total or* n *blueberries total. Find* $n - m$.

Problem 3 *Suppose that* $a_1 = 1$, $a_2 = 2$, *and for any* $n \geq 3$, $a_n = a_1 + a_2 + \cdots + a_{n-1}$. *Find* $\frac{a_{2021}}{a_{2020}}$.

Problem 4 *Cat and Claire are having a conversation about Cat's favorite number.*

Cat says, "My favorite number is a two-digit positive integer that is the product of three distinct prime numbers!"

Claire says, "I don't know your favorite number yet, but I do know that among four of the numbers that might be your favorite number, you could start with any one of them, add a second, subtract a third, and get the fourth!"

Cat says, "That's cool! My favorite number is not among those four numbers, though."

Claire says, "Now I know your favorite number!"

What is Cat's favorite number?

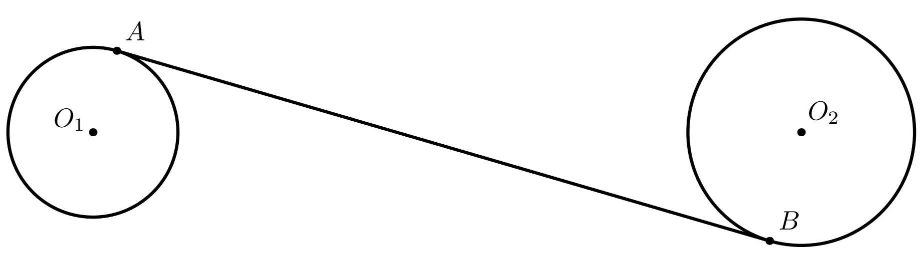
Problem 5 *Let* $ATHEM$ *be a convex pentagon with* $AT = 14$, $TH = MA = 20$, $HE = EM = 15$, *and* $\angle THE = \angle EMA = 90°$. *Find the area of* $ATHEM$.

Problem 6 *Kara rolls a six-sided die six times, and notices that the results satisfy the following conditions:*

- *She rolled a* 6 *exactly three times;*
- *The product of her first three rolls is the same as the product of her last three rolls.*

How many distinct sequences of six rolls could Kara have rolled?

Problem 7 *Suppose two circles* Ω_1 *and* Ω_2 *with centers* O_1 *and* O_2 *have radii* 3 *and* 4, *respectively. Suppose that points* A *and* B *lie on circles* Ω_1 *and* Ω_2, *respectively, such that segments* AB *and* O_1O_2 *intersect and that* AB *is tangent to* Ω_1 *and* Ω_2. *If* $O_1O_2 = 25$, *find the area of quadrilateral* O_1AO_2B.



Problem 8 *Let A and B be digits between 0 and 9, and suppose that the product of the two-digit numbers $\overline{AB}$ and $\overline{BA}$ is equal to k. Given that $k+1$ is a multiple of 101, find k.*

Problem 9 *Ali defines a pronunciation of any sequence of English letters to be a partition of those letters into substrings such that each substring contains at least one vowel. For example, A | THEN | A, ATH | E | NA, ATHENA, and AT | HEN | A are all pronunciations of the sequence ATHENA. How many distinct pronunciations does YALEMATHCOMP have? (Y is not a vowel.)*

Problem 10 *Suppose that $a_1, a_2, a_3, \ldots$ is an infinite geometric sequence such that for all $i \geq 1$, a_i is a positive integer. Suppose furthermore that $a_{20} + a_{21} = 20^{21}$. If the minimum possible value of a_1 can be expressed as $2^a 5^b$ for positive integers a and b, find $a+b$.*

Problem 11 *A right rectangular prism has integer side lengths a, b, and c. If $lcm(a,b) = 72$, $lcm(a,c) = 24$, and $lcm(b,c) = 18$, what is the sum of the minimum and maximum possible volumes of the prism?*

Problem 12 *Let Γ_1 and Γ_2 be externally tangent circles with radii lengths 2 and 6, respectively, and suppose that they are tangent to and lie on the same side of line ℓ. Points A and B are selected on ℓ such that Γ_1 and Γ_2 are internally tangent to the circle with diameter AB. If $AB = a + b\sqrt{c}$ for positive integers a, b, c with c squarefree, then find $a+b+c$.*