Math Majors of America Tournament for High Schools

2019 Tiebreaker Test

Name: __________

ID : __________

Team : __________

– 75 minutes
– no calculators
– show reasoning

1. S is a set of positive integers with the following properties:

(a) There are exactly 3 positive integers missing from S.

(b) If a and b are elements of S, then $a + b$ is an element of S. (We allow a and b to be the same.)

Find all possibilities for the set S (with proof).

2. In the trapezoid $ABCD$, both $\angle B$ and $\angle C$ are right angles, and all four sides of the trapezoid are tangent to the same circle. If $\overline{AB} = x$ and $\overline{CD} = y$, find the area of $ABCD$ (with proof).

3. Let m and n be positive integers. Alice wishes to walk from the point $(0, 0)$ to the point (m, n) in increments of $(1, 0)$ and $(0, 1)$, and Bob wishes to walk from the point $(0, 1)$ to the point $(m, n + 1)$ in increments of $(1, 0)$ and $(0, 1)$. Find (with proof) the number of ways for Alice and Bob to get to their destinations if their paths never pass through the same point (even at different times).

4. The continuous function $f(x)$ satisfies $c^2 f(x + y) = f(x)f(y)$ for all real numbers x and y, where $c > 0$ is a constant. If $f(1) = c$, find $f(x)$ (with proof).