



Math Majors of America Tournament for High Schools 2019 Individual Test

1. When Charles traveled from Hawaii to Chicago, he moved his watch 5 hours backwards instead of 5 hours forwards. He plans to wake up at 7:00 the next morning (Chicago time). When he wakes up during the night and sees that his watch says 6:00, how many more hours should he sleep? (He has a 12-hour watch, not a 24-hour watch.)

2. Rover's dog house in the middle of a large grassy yard is a regular hexagon with side length 10. His leash, which has length 20, connects him to one vertex on the outside of the dog house. His leash cannot pass through the interior of the dog house. What is the total area of the yard (i.e., outside the doghouse) that he can roam? (Give your answer in units squared.)

3. Daniel rolls three fair six-sided dice. Given that the sum of the three numbers he rolled was 6, what is the probability that all of the dice showed different numbers?

4. The points A , B , and C lie on a circle centered at the point O . Given that $m\angle AOB = 110^\circ$ and $m\angle CBO = 36^\circ$, there are two possible values of $m\angle CAO$. Give the (positive) difference of these two possibilities (in degrees).

5. Joanne has four piles of sand, which weigh 1, 2, 3, and 4 pounds, respectively. She randomly chooses a pile and distributes its sand evenly among the other three piles. She then chooses one of the remaining piles and distributes its sand evenly among the other two. What is the expected weight (in pounds) of the larger of these two final piles?

6. When $15!$ is converted to base 8, it is expressed as $\overline{230167356abc00}$ for some digits a , b , and c . Find the missing string \overline{abc} .

7. Construct triangles $\triangle ABC$ and $\triangle A'B'C'$ such that $\overline{AB} = 10$, $\overline{BC} = 11$, $\overline{AC} = 12$, C lies on segment $\overline{A'A}$, B lies on $\overline{C'C}$, A lies on $\overline{B'B}$, and $\overline{A'C} = \overline{C'B} = \overline{B'A} = 1$. Find the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$.

8. Given that $x^4 + y^4 + z^4 = 1$, let a be the maximum possible value of $x + y + z$, let b be the minimum possible value of $x + y + z$, let c be the maximum possible value of $x - y - z$, and let d be the minimum possible value of $x - y - z$. What is the value of $abcd$?

9. How many (possibly empty) subsets of $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ do not contain any pair of elements with difference 2?

10. The positive real numbers x and y satisfy $x^2 = y^2 + 72$. If x^2 , y^2 , and $(x + y)^2$ are all integers, what is the largest possible value of $x^2 + y^2$?

11. There are N ways to decompose a regular 2019-gon into triangles (by drawing diagonals between the vertices of the 2019-gon) such that each triangle shares at least one side with the 2019-gon. What is the largest integer a such that 2^a divides N ?

12. Anna has a 5×5 grid of pennies. How many ways can she arrange them so that exactly two pennies show heads in each row and in each column?

Name: _____

Team : _____

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

12. _____

- 75 minutes
- no calculators
- simplify answers