Math Majors of America Tournament for High Schools

2016 Tiebreaker Test

Name: __________

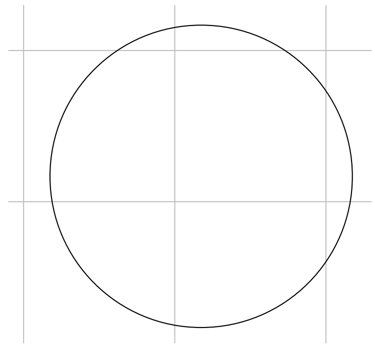
Team : __________

ID: __________

- 75 minutes
- no calculators
- show reasoning
- ID on each page
- 1 problem/page

1. Let *unit blocks* be unit squares in the coordinate plane with vertices at lattice points (points (a, b) such that a and b are both integers). Prove that a circle with area π can cover parts of no more than 9 unit blocks.

The circle below covers part of 8 unit blocks.



2. Suppose we have 2016 points in a 2-dimensional plane such that no three lie on a line. Two quadrilaterals are *not* disjoint if they share an edge or vertex, or if their edges intersect. Show that there are at least 504 quadrilaterals with vertices among these points such that any two of the quadrilaterals *are* disjoint.

3. Show that there are no integers x, y, z, and t such that $\sqrt[3]{x^5 + y^5 + z^5 + t^5} = 2016$.

4. For real numbers a, b, c with $a + b + c = 3$, prove that $a^2(b-c)^2 + b^2(c-a)^2 + c^2(a-b)^2 \geq \frac{9}{2}abc(1 - abc)$ and state when equality is reached.

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Jane Street