Mathathon Round 1 (2 points each)

1. This year, the Mathathon consists of 7 rounds, each with 3 problems. Another math test, Aspartaime, consists of 3 rounds, each with 5 problems. How many more problems are on the Mathathon than on Aspartaime?

2. Let the solutions to $x^3 + 7x^2 - 242x - 2016 = 0$ be a, b, and c. Find $a^2 + b^2 + c^2$. (You might find it helpful to know that the roots are all rational.)

3. For triangle ABC, you are given $AB = 8$ and $\angle A = 30°$. You are told that BC will be chosen from amongst the integers from 1 to 10, inclusive, each with equal probability. What is the probability that once the side length BC is chosen there is exactly one possible triangle ABC?

Mathathon Round 2 (3 points each)

4. It's raining! You want to keep your cat warm and dry, so you want to put socks, rain boots, and plastic bags on your cat's four paws. Note that for each paw, you must put the sock on before the boot, and the boot before the plastic bag. Also, the items on one paw do not affect the items you can put on another paw. How many different orders are there for you to put all twelve items of rain footwear on your cat?

5. Let a be the square root of the least positive multiple of 2016 that is a square. Let b be the cube root of the least positive multiple of 2016 that is a cube. What is $a - b$?

6. Hypersomnia Cookies sells cookies in boxes of 6, 9 or 10. You can only buy cookies in whole boxes. What is the largest number of cookies you cannot exactly buy? (For example, you couldn't buy 8 cookies.)

Mathathon Round 3 (4 points each)

7. There is a store that sells each of the 26 letters. All letters of the same type cost the same amount (i.e. any 'a' costs the same as any other 'a'), but different letters may or may not cost different amounts. For example, the cost of spelling "trade" is the same as the cost of spelling "tread," even though the cost of using a 't' may be different from the cost of an 'r.' If the letters to spell out 1 cost \$1001, the letters to spell out 2 cost \$1010, and the letters to spell out 11 cost \$2015, how much do the letters to spell out 12 cost?

8. There is a square $ABCD$ with a point P inside. Given that $PA = 6$, $PB = 9$, $PC = 8$. Calculate PD.

9. How many ordered pairs of positive integers (x, y) are solutions to $x^2 - y^2 = 2016$?

Mathathon Round 4 (6 points each)

10. Given a triangle with side lengths 5, 6 and 7, calculate the sum of the three heights of the triangle.

11. There are 6 people in a room. Each person simultaneously points at a random person in the room that is not him/herself. What is the probability that each person is pointing at someone who is pointing back to them?

12. Find all x such that $\sum_{i=0}^{\infty} ix^i = \frac{3}{4}$

. . . . Mathathon Round 5 (7 points each) . . . .

13. Let $\{a\}_{n\geq 1}$ be an arithmetic sequence, with $a_1 = 0$, such that for some positive integers k and x we have $a_{k+1} = \binom{k}{x}$, $a_{2k+1} = \binom{k}{x+1}$, and $a_{3k+1} = \binom{k}{x+2}$. Let $\{b\}_{n\geq 1}$ be an arithmetic sequence of integers with $b_1 = 0$. Given that there is some integer m such that $b_m = \binom{k}{x}$, what is the number of possible values of b_2?

14. Let $A = \arcsin(\frac{1}{4})$ and $B = \arcsin(\frac{1}{7})$. Find $\sin(A+B)\sin(A-B)$.

15. Let $\{f_i\}_{i=1}^{9}$ be a sequence of continuous functions such that $f_i : \mathbb{R} \to \mathbb{Z}$ is continuous (i.e. each f_i maps from the real numbers to the integers). Also, for all i, $f_i(i) = 3^i$. Compute $\sum_{k=1}^{9} f_k \circ f_{k-1} \circ \cdots \circ f_1(3^{-k})$.

. . . . Mathathon Round 6 (8 points each) . . . .

16. If x and y are integers for which $\dfrac{10x^3 + 10x^2y + xy^3 + y^4}{203} = 1134341$ and $x - y = 1$, then compute $x + y$.

17. Let T_n be the number of ways that n letters from the set $\{a, b, c, d\}$ can be arranged in a line (some letters may be repeated, and not every letter must be used) so that the letter a occurs an odd number of times. Compute the sum $T_5 + T_6$.

18. McDonald plays a game with a standard deck of 52 cards and a collection of chips numbered 1 to 52. He picks 1 card from a fully shuffled deck and 1 chip from a bucket, and his score is the product of the numbers on card and on the chip. In order to win, McDonald must obtain a score that is a positive multiple of 6. If he wins, the game ends; if he loses, he eats a burger, replaces the card and chip, shuffles the deck, mixes the chips, and replays his turn. The probability that he wins on his third turn can be written in the form $\frac{x^2 \cdot y}{z^3}$ such that x, y, and z are relatively prime positive integers. What is $x + y + z$? (NOTE: Use Ace as 1, Jack as 11, Queen as 12, and King as 13)

. . . . Mathathon Round 7 (9 points each) . . . .

19. Let $f_n(x) = \ln(1 + x^{2^n} + x^{2^{n+1}} + x^{3\cdot 2^n})$. Compute $\sum_{k=0}^{\infty} f_{2k}\left(\frac{1}{2}\right)$.

20. $ABCD$ is a quadrilateral with $AB = 183$, $BC = 300$, $CD = 55$, $DA = 244$, and $BD = 305$. Find AC.

21. Define $\overline{xyz(t+1)} = 1000x + 100y + 10z + t + 1$ as the decimal representation of a four digit integer. You are given that $3^x5^y7^z2^t = \overline{xyz(t+1)}$ where x, y, z, and t are non-negative integers such that t is odd and $0 \leq x, y, z, (t+1) \leq 9$. Compute $3^x5^y7^z$.