Individual Round

Girls in Math at Yale

February 1, 2020

Problem 1 *If $3a + 1 = b$ and $3b + 1 = 2020$, what is a?*

Problem 2 *Tracy draws two triangles: one with vertices at $(0, 0)$, $(2, 0)$, and $(1, 8)$ and another with vertices at $(1, 0)$, $(3, 0)$, and $(2, 8)$. What is the area of overlap of the two triangles?*

Problem 3 *If p, q, and r are prime numbers such that $p + q + r = 50$, what is the maximum possible value of pqr?*

Problem 4 *Points A, B, C, D lie on a circle of radius 4 such that $BC = 8$, $BD = 4$, and $m\angle ABC = 27°$. If segments $\overline{AB}$ and $\overline{CD}$ do not intersect, what is the value of $m\angle ACD$? (Give your answer in degrees.)*

Problem 5 *Express $\sqrt{14 - \sqrt{52}} - \sqrt{14 + \sqrt{52}}$ as a rational number.*

Problem 6 *Let $a_0 = 1$, and let $a_n = 1 + \frac{1}{a_{n-1}}$ for every integer $n \geq 1$. Find the value of the product $a_1 a_2 \cdots a_9$.*

Problem 7 *Miki wants to distribute 75 identical candies to the students in her class such that each students gets at least 1 candy. For what number of students does Miki have the greatest number of possible ways to distribute the candies?*

Problem 8 *Let $ABCD$ be a rectangle. Let points E, F, G, and H lie on the segments $\overline{AB}$, $\overline{AD}$, $\overline{BC}$, and $\overline{CD}$ (respectively) such that both $\overline{EF}$ and $\overline{GH}$ are parallel to $\overline{BD}$. If $\triangle AFE$ is congruent to $\triangle BEG$ and $\frac{AE}{HC} = \frac{1}{2}$, what is $\frac{AB}{BC}$?*

Problem 9 *If $a_1, a_2, a_3, \ldots$ is a geometric sequence satisfying*

$$\frac{19a_{2019} + 19a_{2021}}{a_{2020}} = 25 + \frac{6a_{2006} + 6a_{2010}}{a_{2008}}$$

and $0 < a_1 < a_2$, what is the value of $\frac{a_2}{a_1}$?

Problem 10 *Elizabeth has an infinite grid of squares. (Each square is next to the four squares directly above it, below it, to its left, and to its right.) She colors in some of the squares such that the following two conditions are met: (1) no two colored squares are next to each other; (2) each uncolored square is next to exactly one colored square. In a 20×20 subgrid of this infinite grid, how many colored squares are there?*

Problem 11 *Find the smallest whole number $N \geq 2020$ such that N has twice as many even divisors as odd divisors and N^2 has a remainder of 1 when it is divided by 15.*

Problem 12 *We say that the sets A, B, and C form a "sunflower" if $A \cap B = A \cap C = B \cap C$. ($A \cap B$ denotes the intersection of the sets A and B.) If A, B, and C are independently randomly chosen 4-element subsets of the set $\{1, 2, 3, 4, 5, 6\}$, what is the probability that A, B, and C form a sunflower?*