



Math Majors of America Tournament for High Schools 2016 Mixer Test

1. Give a fake proof that $0 = 1$ on the back of this page. The most convincing answer to this question at this test site will receive a point.

2. It is often said that once you assume something false, anything can be derived from it. You may assume for this question that $0 = 1$, but you can only use other statements if they are generally accepted as true or if you prove them from this assumption and other generally acceptable mathematical statements. With this in mind, on the back of this page prove that every number is the same number.

1. write on back
2. write on back
3. _____
4. _____
5. _____
6. _____
7. _____
8. write on back
9. _____
10. _____
11. _____
12. _____

3. Suppose you write out all integers between 1 and 1000 inclusive. (The list would look something like $1, 2, 3, \dots, 10, 11, \dots, 999, 1000$.) Which digit occurs least frequently?

4. Pick a real number between 0 and 1 inclusive. If your response is r and the standard deviation of all responses at this site to this question is σ , you will receive $r(1 - (r - \sigma)^2)$ points.

5. Find the sum of all possible values of x that satisfy $243^{x+1} = 81^{x^2+2x}$.

6. How many times during the day are the hour and minute hands of a clock aligned?

7. A group of $N + 1$ students are at a math competition. All of them are wearing a single hat on their head. N of the hats are red; one is blue. Anyone wearing a red hat can steal the blue hat, but in the process that person's red hat disappears. In fact, someone can only steal the blue hat if they are wearing a red hat. After stealing it, they would wear the blue hat. Everyone prefers the blue hat over a red hat, but they would rather have a red hat than no hat at all. Assuming that everyone is perfectly rational, find the largest prime N such that nobody will ever steal the blue hat.

8. On the back of this page, prove there is no function $f(x)$ for which there exists a (finite degree) polynomial $p(x)$ such that $f(x) = p(x)(x + 3) + 8$ and $f(3x) = 2f(x)$.

9. Given a cyclic quadrilateral $YALE$ with $YA = 2, AL = 10, LE = 11, EY = 5$, what is the area of $YALE$?

10. About how many pencils are made in the U.S. every year? If your answer to this question is p , and our (good) estimate is ρ , then you will receive $\max(0, 1 - \frac{1}{2}|\log_{10}(p) - \log_{10}(\rho)|)$ points.

11. The largest prime factor of $520,302,325$ has 5 digits. What is this prime factor?

12. The previous question was on the individual round from last year. It was one of the least frequently correctly answered questions. The first step to solving the problem and spotting the pattern is to divide $520,302,325$ by an appropriate integer. Unfortunately, when solving the problem many people divide it by n instead, and then they fail to see the pattern. What is n ?

- 75 minutes
- no calculators
- simplify answers

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