



Math Majors of America Tournament for High Schools 2016 Individual Test

- For what value of x is the function $f(x) = (x - 2)^2$ minimized?
- Two spheres A and B have centers $(0, 0, 0)$ and $(2016, 2016, 1008)$. Sphere A has a radius of 2017. If A and B are externally tangent, what is the radius of sphere B ?
- Consider a white, solid cube of side length 5 made of $5 \times 5 \times 5 = 125$ identical unit cubes with faces parallel to the faces of the larger cube. The cube is submerged in blue paint until the entire exterior of the cube is painted blue, so that a face of a smaller cube is blue if and only if it is part of a face of the larger cube. A random smaller cube is selected and rolled. What is the probability that the up-facing side is blue?
- [This problem was thrown out.]
- Find the largest prime factor of 4003997, given that 4003997 is the product of two primes.
- Elaine is writing letters to six friends. She has six addressed letters and six addressed envelopes. She puts each letter randomly into an envelope without first checking the name on the envelope. What is the probability that exactly one envelope has the correct letter?
- Colin has written the numbers $1, 2, \dots, n$ on a chalk board. He will erase at most 4 of the numbers (he might choose not to erase any of the numbers) and then circle $n - 4$ of the remaining numbers. There are exactly 2016 possible ways to do this. Find n . (You should assume that circling the same set of numbers but erasing different numbers should count as different possible ways.)
- Let $R_n = r_1, r_2, r_3, \dots, r_n$ be a finite sequence of integers such that for all possible i , r_i is either $-1, 0, 1$ or 2 . Furthermore, for all i such that $1 \leq i < n$, r_i and r_{i+1} have opposite parity (i.e. one is odd and the other is even). Finally, -1 and 2 do not occur adjacently in the sequence. Given that r_1 must be even (i.e. either $r_1 = 0$ or $r_1 = 2$), $S(n)$ is the number of possible sequences R_n could be. For example, $S(1) = 2$. For what k is $S(k) = 12^2$?
- What is the largest prime p for which the numbers $p^2 - 8$, $p^2 - 2$, and $p^2 + 10$ are all prime as well?
- Express $\sqrt{25 + 21 \cdot 22 \cdot 23 \cdot 24 \cdot 25}$ as an integer.
- Let $ABCD$ be a quadrilateral where AC bisects $\angle A$, $AB \neq AD$, $BC = CD = 7$, and $AC \cdot BD = 36$. Find $AB + AD$.
- Find the largest integer x for which there is an integer y such that $x^4 + 12x^3 + 39x^2 + 17x - 57 = y^3$.

Name: _____

Team : _____

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

12. _____

– 75 minutes
– no calculators
– simplify answers

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